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# Josephson current transport through T-shaped double quantum dots

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## Abstract

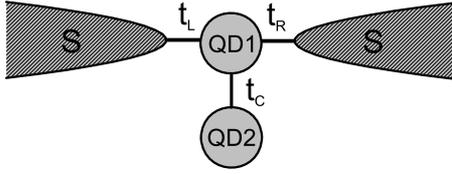
The Josephson current through T-shaped double quantum dots (TDQD) has been investigated theoretically using the non-equilibrium Green's function method. Hybridization of energy levels of the central quantum dot (QD) and side QD results in the renormalization of Andreev bound states, which dominates the Josephson current. Consequently, Josephson critical current can be modulated to interference construction and destruction by adjusting the energy levels of two QDs  $\varepsilon_1$ ,  $\varepsilon_2$ , and interdot coupling  $t_c$ . In detail, when  $\varepsilon_1\varepsilon_2 = t_c^2$  is fulfilled, interference construction occurs and when  $\varepsilon_2 = 0$ , interference destruction happens. These results are similar to the interference behavior of linear conductance in normal TDQD devices. In addition, Josephson critical current also shows Fano characteristics with variation of  $\varepsilon_2$  and the resonance line shape is determined by  $\varepsilon_1$ . Finally, the Josephson current has a symmetric relation of  $I(\varepsilon_1, \varepsilon_2) = I(-\varepsilon_1, -\varepsilon_2)$  due to electron-hole symmetry.

## 1. Introduction

Owing to advances in nanotechnology, the interference of the transport of phase coherent electrons in mesoscopic devices has been widely researched in recent years [1–3]. It provides detectable quantum states within devices which facilitate novel fabrication of quantum apparatus. The Aharonov–Bohm (AB) interferometer, a typical interferometer, is more controllable if a quantum dot (QD) is embedded in one arm or each arm. An AB interferometer with two QDs inserted in two arms is useful for detecting electron interference and possible realization of two-electron spin entanglement [4]. In the situation of just one QD in one arm of an AB interferometer, the Fano effect, first proposed in atomic physics [5], can be observed and modulated by changing magnetic flux  $\Phi$  through the ring or the energy level of the QD  $\varepsilon_d$ . The Fano effect arises from the interference between a continuum energy spectrum and discrete energy states. It shows a typical Fano resonance of transmission probability  $T(E)$  as a function of the discrete energy level  $\varepsilon_d$  [6, 7]. Transmission probability is generally of the type  $T(E) \propto \frac{(\epsilon+q)^2}{\epsilon^2+1}$ , in which  $\epsilon = \frac{E-\varepsilon_d}{\Gamma}$  and  $\Gamma$  is the coupling strength between QD and the leads. The Fano parameter  $q$  is generally a complex number which counts the characteristics of the corresponding model, and it determines the line shape of the resonance. What is more, the Fano effect

in mesoscopic devices has been realized in experiments [8, 9] and has evoked other related topics such as the Kondo–Fano effect [10].

Similar to the Fano model of a QD in an AB interferometer, a QD with another QD side couple to it (as shown in figure 1), called the T-shaped double QD (TDQD) model, is another prototype of the Fano model. In this model, compared with the side QD which is not connected directly to the leads, the spectrum of electrons in the central QD connected to a source and a drain act as the continuum energy spectrum. Interference occurs by electron transport through two paths, one is tunneling through the central QD and the other is through the central QD and then with extra scattering by the side QD. The energy level of the side QD plays the role of a Fano resonance shape modulator, because an extra phase  $\Theta_{\text{QD}}$  emerges when the electron is elastically scattered by the side QD [11].  $\Theta_{\text{QD}}$  is zero when the energy level of the side QD  $\varepsilon_2 = 0$  and is  $\pi/2$  ( $-\pi/2$ ) when  $\varepsilon_2 \gg 0$  ( $\varepsilon_2 \ll 0$ ). So transport enhancement and suppression can be realized by adjusting the QDs' levels. Electron transport through the normal(N)-TDQD-N model has been widely researched. Güçlü and coworkers studied the TDQD when the side QD is a Kondo impurity and found suppression of the conductance [12]. Wu *et al* focused on the TDQD consisting of a central Kondo dot and a side coupled



**Figure 1.** Schematic diagram for the TDQD system. QD1 is connected to two superconductor leads with coupling parameter  $t_L$  and  $t_R$ . QD2 is side coupled to QD1 with coupling parameter  $t_c$ .

noninteracting QD, and they found that linear conductance of Kondo unitary is broken down by the side QD [13, 14]. Cornaglia and Grepel showed that when both QDs are in the Kondo regime, conductance is controlled by interdot coupling and the Kondo temperature of QDs [15]. Tanamoto and Nishi researched the modulation of the Fano dip in a similar model with a side coupled QD molecular model [16]. Previous works all concentrate on the TDQD device coupled to normal contacts, while Josephson current through TDQD has not been investigated yet. When a TDQD is embedded between two superconductor (S) leads, the Cooper pairs could tunnel through the TDQD device even in the zero bias case because of the superconductor phase difference of the two leads. Interference of the Cooper pairs passing through the two paths should also happen. The interference construction and destruction, Fano effect of the Cooper pair transport, as well the characteristics of the Andreev bound states are the motivations behind this work.

In this paper we consider a TDQD structure connected to two superconductor leads. By using the non-equilibrium Green's function method, the Josephson current expression and Andreev bound states are obtained. The Andreev bound states are strongly affected by the side QD due to hybridization of two QD levels  $\varepsilon_1$  and  $\varepsilon_2$ . While in weak coupling to the superconductor leads, the positions of the Andreev bound states are close to the molecular levels of isolated DQDs. The critical Josephson current  $I_c$  can be modulated to interference construction and destruction due to the interference of Cooper pair transport through two paths. In detail, when  $\varepsilon_2 = 0$ ,  $I_c$  is suppressed and when relation  $t_c^2 = \varepsilon_1 \varepsilon_2$  is fulfilled, interference construction of Josephson current occurs. Fano type resonance of  $I_c$  is observed by adjusting  $\varepsilon_2$  with the line shape depending on  $\varepsilon_1$ . When  $\varepsilon_1 \neq 0$ , typical asymmetric Fano resonance of  $I_{c-\varepsilon_2}$  is found, whereas when  $\varepsilon_1 = 0$ ,  $I_{c-\varepsilon_2}$  is symmetric. Finally we found that the Josephson current has the property,  $I(\varepsilon_1, \varepsilon_2) = I(-\varepsilon_1, -\varepsilon_2)$ , which includes the electron-hole symmetry.

The rest of the paper is organized as follows. In section 2, the Hamiltonian and Josephson current expressions are presented. In section 3, we show our main numerical results of the Josephson current-superconducting phase relation, the Andreev bound states phase relation, the interference construction and destruction of critical current related to QD levels, and the Fano characteristics of the critical current. Finally a brief conclusion is given in section 4.

## 2. Model and formulations

We consider a TDQD structure connected to two Bardeen-Cooper-Schrieffer (BCS) superconductor leads as shown in figure 1. QD1 is connected to both superconductor leads with coupling parameters  $t_L$  and  $t_R$  respectively. QD2 is side coupled to QD1 with interdot coupling parameter  $t_c$ . The Hamiltonian of the system can be written as

$$H = \sum_{\alpha=L,R} H_{\alpha} + \sum_{i=1,2} H_i + H_t, \quad (1)$$

where  $H_{\alpha}$  and  $H_i$  are the Hamiltonian of the  $\alpha$ th superconductor lead and the  $i$ th QD, respectively.  $H_t$  is the tunneling term, including the coupling of QD1 to two superconductor leads and coupling between QD1 and QD2. Terms in equation (1) are expressed as:

$$\begin{aligned} H_{\alpha} &= \sum_{k\sigma} \varepsilon_k C_{k\sigma,\alpha}^{\dagger} C_{k\sigma,\alpha} \\ &+ \sum_k \Delta (C_{k\downarrow,\alpha} C_{-k\uparrow,\alpha} + C_{-k\uparrow,\alpha}^{\dagger} C_{k\downarrow,\alpha}^{\dagger}) \\ H_i &= \sum_{\sigma} \varepsilon_i d_{i\sigma}^{\dagger} d_{i\sigma} \\ H_t &= \sum_{k,\sigma,\alpha} (t_{\alpha} e^{\frac{i\theta_{\alpha}}{2}} C_{k\sigma,\alpha}^{\dagger} d_{1\sigma} + t_{\alpha} e^{-\frac{i\theta_{\alpha}}{2}} d_{1\sigma}^{\dagger} C_{k\sigma,\alpha}) \\ &+ t_c (d_{1\sigma}^{\dagger} d_{2\sigma} + d_{2\sigma}^{\dagger} d_{1\sigma}), \end{aligned} \quad (2)$$

where  $\Delta$  and  $\theta_{\alpha}$  are the superconductor energy gap and phase. Here we have taken a unitary transformation as [17], so the superconductor phase  $\theta_{\alpha}$  emerges in the tunneling Hamiltonian  $H_t$  in equation (2). We consider single level QDs, and  $\varepsilon_i$  is the energy level of the  $i$ th QD. Here the electron-electron interaction in the QDs is neglected, because we consider the large QD. In fact, if the temperature is higher than the Kondo temperature, the electron-electron interaction is only to widen the space of the levels, and the results are qualitatively the same.

The current through the  $\alpha$ th lead is calculated from the evolution of the electron number operator  $N_{\alpha} = \sum_{k\sigma} C_{k\sigma,\alpha}^{\dagger} C_{k\sigma,\alpha}$  [18, 19],

$$\begin{aligned} I_{\alpha} &= -e \langle \dot{N} \rangle \\ &= \frac{4e}{\hbar} \text{Re} \int \frac{dE}{2\pi} t_{\alpha} e^{\frac{i\theta_{\alpha}}{2}} G_{1\alpha,11}^{<}(E) \end{aligned} \quad (3)$$

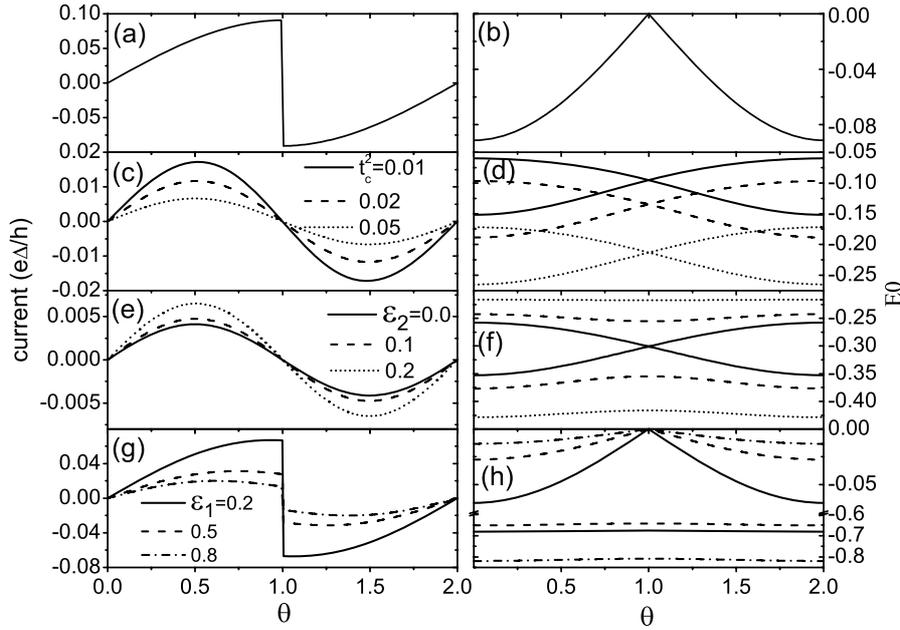
$G_{1\alpha}^{<}(E)$  is the Fourier transformation of  $\mathbf{G}_{1\alpha}^{<}(t-t')$ , and

$$\mathbf{G}_{1\alpha}^{<}(t-t') \equiv i \sum_k \begin{pmatrix} \langle C_{k\uparrow,\alpha}^{\dagger}(t') d_{1\uparrow}(t) \rangle & \langle C_{-k\downarrow,\alpha}(t') d_{1\uparrow}(t) \rangle \\ \langle C_{k\uparrow,\alpha}^{\dagger}(t') d_{1\downarrow}^{\dagger}(t) \rangle & \langle C_{-k\downarrow,\alpha}(t') d_{1\downarrow}^{\dagger}(t) \rangle \end{pmatrix}$$

under the Nambu representation.

We consider here the dc Josephson effect, thus  $\mathbf{G}_{1\alpha}^{<}(E)$  can be simplified by the fluctuation-dissipation theorem that  $\mathbf{G}_{1\alpha}^{<} = -f(E)(\mathbf{G}_{1\alpha}^r(E) - \mathbf{G}_{1\alpha}^a(E))$  and  $f(E)$  is the Fermi-Dirac distribution and  $\mathbf{G}_{1\alpha}^{r,a}(E)$  are the retarded and advanced Green's functions.

By using Dyson's equation, the retarded Green's function  $\mathbf{G}_{1\alpha}^r(E)$  can be expressed as  $\mathbf{G}_{1\alpha}^r = \mathbf{G}_1^r \mathbf{t}_{\alpha}^* \mathbf{g}_{\alpha}^r$ , and the Green's function  $\mathbf{G}_1^r$  of QD1 is  $\mathbf{G}_1^r = (\mathbf{g}_1^{-1} - \Sigma^r)^{-1}$ .  $\Sigma^r = \mathbf{t}_L^* \mathbf{g}_L^r \mathbf{t}_L + \mathbf{t}_R^* \mathbf{g}_R^r \mathbf{t}_R + \mathbf{t}_c^* \mathbf{g}_2^r \mathbf{t}_c$  is the retarded self-energy



**Figure 2.** Left panel: Josephson current  $I$  versus superconductor phase  $\theta$  for various parameters, (a)  $t_c = 0$ ,  $\Gamma = 0.1$  and  $\varepsilon_1 = 0$ ; (c)  $\varepsilon_1 = \varepsilon_2 = 0$  and  $\Gamma = 0.1$  for different QD coupling  $t_c$ ; (e)  $\varepsilon_1 = 0$ ,  $\Gamma = t_c^2 = 0.1$  for different  $\varepsilon_2$  values; (g)  $\Gamma = t_c^2 = 0.1$  and  $\varepsilon_1, \varepsilon_2$  satisfies  $\varepsilon_1 \varepsilon_2 = t_c^2$ . Right panel: Andreev bound states versus superconductor phase  $\theta$  with the parameters corresponding to the left panel.

coupled to superconductor leads and QD2. Here  $\mathbf{g}_\alpha^r(E)/\mathbf{g}_i^r(E)$  is the Green's function of the isolated superconductor lead or the Green's function of the isolated  $i$ th QD, and  $\mathbf{t}_\alpha/t_c$  is the tunneling matrix corresponding to  $t_\alpha/t_c$  in the Nambu representation. The expression of  $\mathbf{g}_\alpha^r(E)$  is [20]  $\mathbf{g}_\alpha^r(E) = -\pi\rho(E)\begin{pmatrix} \beta(E) & \beta_0(E) \\ \beta_0(E) & \beta(E) \end{pmatrix}$  and  $\mathbf{g}_i^r(E) = \begin{pmatrix} 1/(E - \varepsilon_i + i\eta) & 0 \\ 0 & 1/(E + \varepsilon_i + i\eta) \end{pmatrix}$ , where  $\rho(E)$  is the normal density of states of the superconductor lead,  $\beta_0(E) = \beta\Delta/E$ , and  $\beta(E) = E/\sqrt{\Delta^2 - E^2}$  while  $|E| < \Delta$  and  $\beta(E) = i|E|/\sqrt{E^2 - \Delta^2}$  while  $|E| > \Delta$ . Tunneling matrices are  $\mathbf{t}_\alpha = t_\alpha \begin{pmatrix} e^{i\theta_\alpha/2} & 0 \\ 0 & -e^{-i\theta_\alpha/2} \end{pmatrix}$  and  $\mathbf{t}_c = t_c \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ . For convenience we take the symmetric barriers with  $t_L = t_R$  and  $\theta_L = -\theta_R = \theta/2$ .

Finally we have the reduced Josephson current expression

$$I_\alpha = \frac{-2e}{h} \int \frac{dE}{2\pi} f(E) \frac{\beta_0^2 \Gamma^2 \sin \theta}{\text{Im}\{B\}}, \quad (4)$$

where  $B = (E - \frac{Et_c^2}{E^2 - \varepsilon_1^2} + \Gamma\beta)^2 - (\varepsilon_1 + \frac{\varepsilon_2 t_c^2}{E^2 - \varepsilon_2^2})^2 - \Gamma^2 \beta_0^2 \cos^2 \frac{\theta}{2}$ , and the linewidth function  $\Gamma \equiv 2\pi\rho t_\alpha^2$  describes the coupling strength of QD1 to the superconductor leads, which is assumed independent of the energy  $E$ .

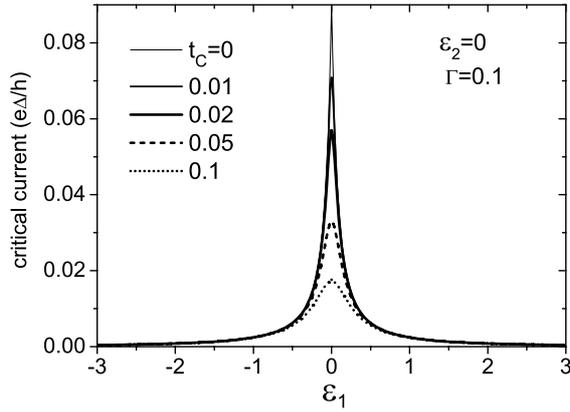
Two parts contribute to the Josephson current, the continuous part  $I_{\text{con}}$  arises from electrons of energy  $E$  outside the superconducting gap  $\Delta$  and the discrete part  $I_{\text{dis}}$  from electrons of energy within the gap  $\Delta$ . The continuous part  $I_{\text{con}}$  is obtained directly by the integral in equation (4), while the discrete part is approached by solving poles of factor  $B$  which are the Andreev bound states. Affected by the QD2, instead of one pair of bound states  $\pm E_0$  in a S-QD-S junction, there are two pairs of Andreev bound states  $E_{i=1,2}^\pm$  with  $E_i^+ = -E_i^-$  and they all make contributions to the current. Besides, as in the S-QD-S Josephson junction, the current  $I_{\text{dis}}$  contributed

by the Andreev bound states is usually much larger than the continuous part.

### 3. Results and discussion

In this section, we will present the numerical results on Josephson current–superconducting phase relations and the corresponding Andreev bound states–phase relations, the interference construction and destruction of critical current, as well as the Fano characteristics of critical current.

In figure 2 we show the current phase relation ( $I$ – $\theta$ ) for different parameters and the corresponding Andreev bound states' phase relation ( $E_i^\pm$ – $\theta$ ). These two relations are connected by  $I_{\text{dis}} = -\frac{2e}{h} \sum_{i,\pm} f(E_i^\pm) \frac{\partial E_i^\pm}{\partial \theta}$  [21, 22]. Because the bound states within the gap are paired with energy of opposite signs, we only show those in half-interval  $[-\Delta, 0]$ . First, by decoupling QD2, the usual S-QD-S junction is displayed in figure 2(a). When  $\varepsilon_1 = 0$ , the current shows a discontinuous jump at  $\theta = \pi$  and meanwhile the Andreev bound states  $\pm E_0$  degenerate at  $E = 0$  [23–25]. When QD2 coupling is considered,  $I$ – $\theta$  is usually a sinuous line shape and the current is suppressed as  $t_c$  is increased (see figure 2(c)). In this case, the Andreev bound states depart from the Fermi energy  $E_F = 0$  which breaks down the resonance and suppresses the current (see figure 2(d)). In figure 2(e), current for different  $\varepsilon_2$  values is shown. The current is enhanced as  $\varepsilon_2$  is away from the Fermi energy which illustrates that in  $\varepsilon_2 = 0$  current is suppressed. Andreev bound states cross when  $\varepsilon_2 = 0$  at  $\theta = \pi$  and they depart from each other as  $\varepsilon_2$  is away from the Fermi level. Finally in figure 2(g), the current jump at  $\theta = \pi$  appears when the relation  $\varepsilon_1 \varepsilon_2 = t_c^2$  is fulfilled. Corresponding Andreev bound states in this condition display the same behavior as in figure 1(b) where one pair of



**Figure 3.** Critical current versus  $\varepsilon_1$  for different interdot coupling  $t_c$ .

bound states degenerates at  $E = 0$  and the current carried by the bound states changes its sign abruptly [26]. This condition can be analytically obtained from the expression  $B$ . Supposing Andreev bound states degenerate at  $E = 0$ , by solving  $B(E = 0) = 0$ , we easily have

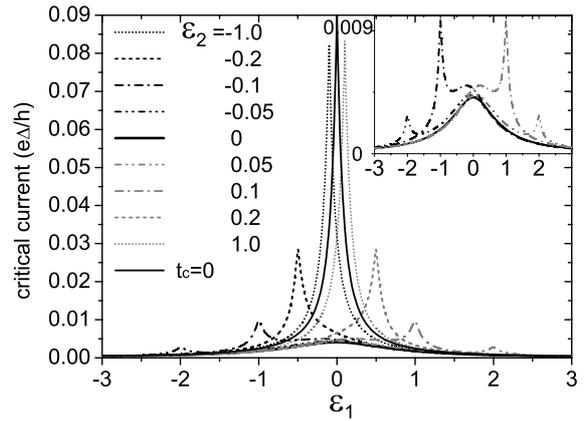
$$\varepsilon_1 \varepsilon_2 = t_c^2, \quad \theta = \pi. \quad (5)$$

This means, if these two conditions hold, that the degeneration of one pair of bound states at the Fermi level will occur and consequently, a Josephson current jump at  $\theta = \pi$  will occur as well. Besides, equation (5) is the condition for maximum critical current, which will be detailed in the following discussions.

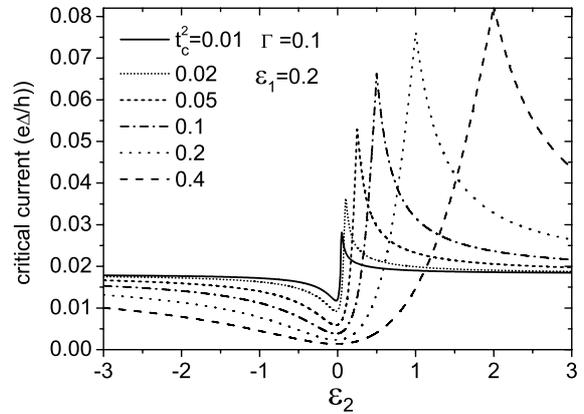
Finally in this  $I$ - $\theta$  discussion, we note that in a N-TDQD-N, the realization of a linear conductance maximum is exactly equation (5). In fact, the values of Andreev bound states in our model when weakly coupled to S leads are very close to the molecular levels of the isolated QD molecule  $\varepsilon^\pm = [(\varepsilon_1 + \varepsilon_2) \pm \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4t_c^2}]/2$  (not shown here) which is also the resonance center of linear conductance in the N-TDQD-N model [13].

Next, we focus on the critical current through the TDQD. The critical current  $I_c$  is obtained by choosing the maximum Josephson current in a  $2\pi$  period of superconductor phase  $\theta$ . In figure 3 we plot  $I_c$  as an energy level of QD1  $\varepsilon_1$  for different interdot coupling  $t_c$  at  $\varepsilon_2 = 0$ .  $I_c$  shows a symmetric peak at  $\varepsilon_1 = 0$ , and this peak is strongly suppressed when  $t_c$  is introduced with even tiny values. This result can be understood from the interference of two paths. When  $\varepsilon_1$  aligns the Fermi level, electrons are easy to transport through the system. However when QD2 is connected with its level  $\varepsilon_2 = E_F = 0$ , electrons being transported through QD1 tend to tunneling into QD2. Then interference destruction between two paths occurs, and the current is decreased. So, while level  $\varepsilon_2$  closes to the Fermi level, QD2 acts as an impurity to scatter the incident electron or Cooper pair. This result resembles the transport through an N-TDQD-N device.

In figure 4, we plot the  $I_c$ - $\varepsilon_1$  relation for different  $\varepsilon_2$ . Here we also show a graph of  $t_c = 0$  for comparison. For given  $t_c$ ,  $I_c$  shows a peak at  $\varepsilon_1 = 0$  when  $\varepsilon_2 = 0$  (see the inset of figure 4). As  $\varepsilon_2$  is moving off the Fermi level, interference construction begins to function. Consequently an extra peak



**Figure 4.** Critical current versus  $\varepsilon_1$  for different  $\varepsilon_2$  values with  $t_c^2 = \Gamma = 0.1$ . The curve for  $t_c = 0$  is also shown for comparison. Inset: enlarged figure of critical current for the curves with  $\varepsilon_2 = 0, \pm 0.05$  and  $\pm 0.1$ .

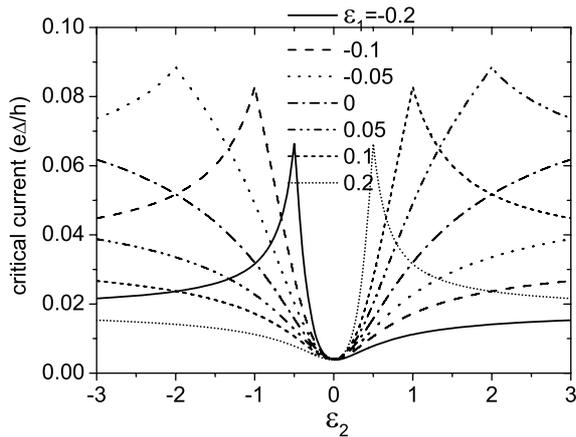


**Figure 5.** Critical current versus  $\varepsilon_2$  for different interdot coupling  $t_c$  for nonzero  $\varepsilon_1$ . Other parameters are  $\varepsilon_1 = 0.2$  and  $\Gamma = 0.1$ .

is shown and the original peak at  $\varepsilon_1 = 0$  becomes obscure. The position of the extra peak is determined by equation (5). When equation (5) is satisfied, one pair of Andreev bound states aligns to the Fermi level which facilitates the transport. With  $\varepsilon_2$  moving further off the Fermi level, or in other words, when QD2 is gradually isolated from QD1, the curve of  $I_c$ - $\varepsilon_1$  tends to that of the S-QD-S junction (i.e. the  $t_c = 0$  case).

Now we focus on the Fano resonance characteristics of critical current. In figure 5 we plot a graph of  $I_c$ - $\varepsilon_2$  for different interdot coupling.  $I_c$ - $\varepsilon_2$  shows a typical Fano asymmetric line shape when  $\varepsilon_1 \neq 0$ . There is an obvious Fano valley at  $\varepsilon_2 = 0$  and a peak depending on equation (5). With enhanced  $t_c$ , the interference destruction and construction are enhanced even more. The Fano valley is still at  $\varepsilon_2 = 0$  with smaller critical current, but the peak is moved away with larger magnitude of the critical current.

Finally, we also found that the Fano line shape of critical Josephson current can be modulated by the QD1 level, which is like the Fano effect in an AB interferometer with its Fano line shape modulated by magnetic flux. In figure 6 we plot  $I_c$ - $\varepsilon_2$  for different  $\varepsilon_1$  values. The curves show a valley at  $\varepsilon_2 = 0$  and a peak at  $\varepsilon_2 = t_c^2/\varepsilon_1$ . In particular, with the change of  $\varepsilon_1$ ,



**Figure 6.** Critical current versus  $\varepsilon_2$  for different  $\varepsilon_1$ . Other parameters are  $t_c^2 = 0.1$  and  $\Gamma = 0.1$ .

the Fano peak can be modulated and the peak position can vary from the right side with  $\varepsilon_2 > 0$  to the left side with  $\varepsilon_2 < 0$ . In addition, a relation  $I_c(\varepsilon_1, \varepsilon_2) = I_c(-\varepsilon_1, -\varepsilon_2)$  is found from figure 6, which reflects the basic physics of electron–hole symmetry. In fact, by taking the transform  $d_i$  to  $\tilde{d}_i^\dagger$  and simultaneously setting the parameters  $(\varepsilon_1, \varepsilon_2)$  to  $(-\varepsilon_1, -\varepsilon_2)$ , the Hamiltonian  $H$  in formula (1) is invariable. So the Josephson current has the relation  $I(\varepsilon_1, \varepsilon_2) = I(-\varepsilon_1, -\varepsilon_2)$ .

#### 4. Summary

The Josephson current through a T-shaped double quantum dot device has been investigated. Josephson critical current can be modulated by energy levels of two QDs  $\varepsilon_1$  and  $\varepsilon_2$ . The interference construction occurs when interdot coupling  $t_c$  and energy levels fulfil  $\varepsilon_1\varepsilon_2 = t_c^2$ , and the interference destruction emerges while  $\varepsilon_2 = 0$ . Critical current versus the side QD level  $\varepsilon_2$  shows Fano characteristics with resonance shape determined by the central QD  $\varepsilon_1$ . In addition, due to electron–hole symmetry, the Josephson current has the relation  $I(\varepsilon_1, \varepsilon_2) = I(-\varepsilon_1, -\varepsilon_2)$ .

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